Effect of Dispersion of a solute in Peristaltic flow of a Jeffrey Fluid

G. Ravi Kiran, G.Radhakrishnamacharya

Abstract— In this paper, the dispersion of a solute in the peristaltic flow of a Jeffrey fluid in the presence of both homogeneous and heterogeneous chemical reactions has been discussed. The average effective dispersion co-efficient has been found using Taylor's limiting condition under long wavelength approximation. It is observed that the average dispersion co-efficient increases with amplitude ratio which implies that dispersion is more in the presence of peristalsis. The average dispersion co-efficient increases with Jeffrey parameter (λ_1) in the cases of both homogeneous and combined homogeneous and heterogeneous chemical reactions. Further, dispersion decreases with homogeneous reaction rate parameter α and heterogeneous reaction rate parameter β.

Index Terms— Dispersion, Irreversible chemical reaction, Jeffrey fluid, Peristalsis.

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1 INTRODUCTION

PERISTALTIC motion is a form of fluid transport that occurs when a progressive wave of area contraction or expansion propagates along the length of a distensible tube or channel containing the fluid. The study of peristaltic flow has received considerable attention during the past few years mainly because of its applications in several disciplines of engineering and biology. Some applications of the peristaltic transport include urine transport from kidney to bladder, chyme movement in the intestines, the movement of spermatozoa in male reproductive tract etc. Also mechanical devices such as dialysis machines, open heart bypass pump machines and infusion pumps use peristalsis. Hence, several authors have studied peristaltic transport of Newtonian fluids under different conditions (Fung and Yih [1], Shapiro et al. [2], Shehawey and Sebaei [3], Nadeem and Akbar [4], [5], Takagi and Balmforth [6]).

Most of bio-fluids such as blood exhibit the behavior of non-Newtonian fluids. Hence, the study of peristaltic transport of non-Newtonian fluids may help to get better understanding of the working of biological systems. Radhakrishnamacharya [7] studied long wavelength approximation to peristaltic motion of a power law fluid. Medhavi [8], Nagarani [9], Ravi Kumar et al. [10] and Narahari and Sreenadh [11] studied peristaltic transport of non-Newtonian fluids under different conditions. Another non-Newtonian fluid that received considerable attention of researchers is Jeffrey fluid, which can be used to represent a physiological fluid. This model is a relatively simpler linear model which uses time derivatives instead of convective derivatives. It represents a rheology different from that of Newtonian fluid. Further, Jeffrey fluid model is significant because Newtonian fluid model can be deduced from this as a special case by taking $\lambda_1 = 0$.

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Some researchers have studied peristaltic motion of a Jeffrey fluid under different conditions. Hayat et al. [12] analyzed three-dimensional flow of Jeffrey fluid. Vajravelu et al. [13] investigated the influence of heat transfer on peristaltic transoport of Jeffrey fluid. Hayat and Ali [14], Kothandapani and Srinivas [15], Nadeem and Akram [16] and Pandey and Tripathi [17] have considered the peristaltic motion of a Jeffrey fluid.

Dispersion describes the spread of particles through random motion from regions of higher concentration to regions of lower concentration. The fluid mechanical aspects of dispersion of a solute also received the attention of some investigators in the last few decades. The dispersion of a solute in a viscous liquid flowing in a circular pipe under laminar conditions was studied by Taylor [18], [19], [20]. Aris [21] while extending Taylor's analysis found that the rate of growth of variance of the solute distribution is proportional to the sum of the molecular diffusion and Taylor diffusion coefficient. In all these investigations, it is assumed that the solute does not chemically react in the liquid in which it is dispersed. However, in a wide variety of problems of chemical engineering, diffusion of a solute takes place with simultaneous chemical reaction in situations such as hydrolysis, gas absorption in an agitated tank, esterification (Padma and Ramana Rao [22]). Hence, Gupta and Gupta [23], Ramana Rao and Padma [24], [25] and Padma and Ramana Rao [22] dealt with the effect of chemical reaction on the dispersion in Newtonian fluids. Dutta et al. [26] and Shukla et al. [27] studied dispersion in non-Newtonian fluids by considering only homogeneous first-order chemical reaction in the bulk of the fluid. Chandra and Agarwal [28] considered dispersion in simple microfluid flows taking only homogeneous reaction into consideration. Philip and Chandra [29] also investigated the effects of heterogeneous and homogeneous reactions on the dispersion of a solute in simple microfluid. Recently, Alemayehu and Radhakrishnamacharya [30], [31] studied the effect of dispersion on peristaltic flow of micropolar and couple stress fluids under different conditions.

Peristalsis and diffusion are very important aspects in biological systems. The effect of dispersion in peristaltic motion of a Jeffrey fluid has not received any attention. It is realized that peristalsis may enhance dispersion of a solute in fluid flow. This, in turn, may help in better absorption of nutrients and drugs in physiological systems. Hence, the study of the interaction of peristalsis with diffusion may lead to better understanding of the flow situation in physiological systems. In the present study, the effect of peristalsis on the dispersion of a solute in a Jeffrey fluid with simultaneous chemical reaction is investigated. Using long wave length approximation and Taylor's approach, closed form solution has been obtained for the dispersion coefficient for both the cases of homogeneous first-order irreversible chemical reaction and combined first-order homogeneous and heterogeneous chemical reactions. The effects of various relevant parameters on the average effective dispersion coefficient are studied.

2 MATHEMATICAL FORMULATION

Consider the dispersion of a solute in peristaltic flow of a Jeffrey fluid in an infinite uniform channel of width 2d and with flexible walls on which are imposed traveling sinusoidal waves of long wavelength. Cartesian coordinate system (x, y) is chosen with x-axis aligned with the center line of the channel. The traveling waves are represented by (Fig. 1)
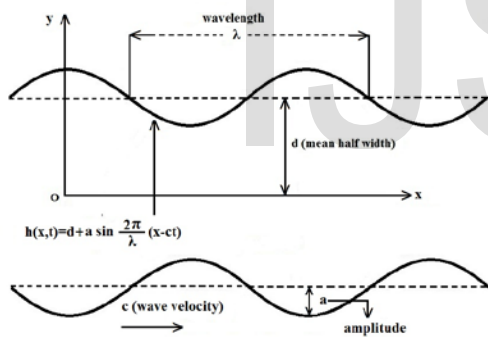

Fig. 1 Geometry of the problem

$$y = \pm h = \pm \left[d + a \sin \frac{2\pi}{\lambda}(x - ct) \right] \tag{1}$$

where a is the amplitude, c is the speed and λ is the wavelength of the peristaltic wave.

The constitutive equations for an incompressible Jeffrey fluid [12], [13], [14], [15], [16], [17] are

$$\overline{T} = -\overline{PI} + \overline{S} \tag{2}$$

$$\overline{S} = \frac{\mu}{1 + \lambda_1}(\dot{\gamma} + \lambda_2 \ddot{\gamma}) \tag{3}$$

where $\overline{T}$, $\overline{S}$ are Cauchy stress tensor and extra stress tensor respectively, $\overline{P}$ is the pressure, $\overline{I}$ is the identity tensor, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time, μ is the dynamic viscosity, $\dot{\gamma}$ is the shear rate and dots over the quantities indicate differentiation with respect to time.

The equations governing two-dimensional motion of an incompressible Jeffrey fluid are reduced to

$$\rho \left[\frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right] u = -\frac{\partial p}{\partial x} + \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} \tag{4}$$

$$\rho \left[\frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right] v = -\frac{\partial p}{\partial y} + \frac{\partial S_{yy}}{\partial y} + \frac{\partial S_{yx}}{\partial x} \tag{5}$$

and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

where u,v are the velocity components in the x and y directions respectively, p is the pressure, ρ is the density and S_{xx}, S_{xy}, S_{yx}, S_{yy} are extra stress components.

Under long wavelength approximation, the governing equations for the present problem reduces to,

$$\frac{\partial p}{\partial x} = \frac{\mu}{1 + \lambda_1} \frac{\partial^2 u}{\partial y^2} \tag{7}$$

$$\frac{\partial p}{\partial y} = 0 \tag{8}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{9}$$

We assume that the walls are inextensible so that only lateral motion takes place and the horizontal displacement of the wall is zero.

Thus, the no-slip boundary condition for the velocity is given by,

$$u = 0 \text{ at } y = \pm h \tag{10}$$

Solving (7) and (8) under the boundary conditions (10), we get

$$u(y) = -\frac{1 + \lambda_1}{\mu} \frac{\partial p}{\partial x} \left[\frac{1}{2}\left(h^2 - y^2 \right) \right] \tag{11}$$

Further, the mean velocity is defined as

$$\overline{u} = \frac{1}{2h} \int_{-h}^{h} u(y)dy \tag{12}$$

Substituting (11) in (12), we get

$$\overline{u} = -\left(\frac{1 + \lambda_1}{\mu} \right) \frac{\partial p}{\partial x} \frac{h^2}{3} \tag{13}$$

If we now consider convection across a plane moving with the mean speed of the flow, then relative to this plane, the fluid velocity is given by

$$u_x = u - \overline{u} \tag{14}$$

Substituting (11) and (13) in (14), we get

$$u_x = -\frac{1}{2}\left(\frac{1 + \lambda_1}{\mu} \right) \frac{\partial p}{\partial x} \left[\frac{h^2}{3} - y^2 \right] \tag{15}$$

2.1 Diffusion with a Homogeneous First order Chemical Reaction

It is assumed that a solute diffuses and simultaneously undergoes a first order irreversible chemical reaction in peri-

staltic transport of Jeffrey fluid in a channel under isothermal conditions. Using Taylor's approximation, i.e., $\dfrac{\partial^2 C}{\partial x^2} \ll \dfrac{\partial^2 C}{\partial y^2}$, the equation for the concentration C of the solute for the present problem is given by

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial y^2} - k_1 C \tag{16}$$

where D is the molecular diffusion co-efficient and k_1 is the first order reaction rate constant.

For typical values of physiologically relevant parameters of this problem, it is realized that $u = c$. Using this condition and making use of the following dimensionless quantities,

$$\theta = \frac{t}{\bar{t}},\ \bar{t} = \frac{\lambda}{u},\ \eta = \frac{y}{d},\ \xi = \frac{(x - \bar{u}t)}{\lambda},\ H = \frac{h}{d} \tag{17}$$

equation (15) reduces to

$$u_x = -\frac{d^2}{2\mu}(1 + \lambda_1)\frac{\partial p}{\partial x}\left[\frac{H^2}{3} - \eta^2\right] \tag{18}$$

and equation (16) becomes,

$$\frac{\partial^2 C}{\partial \eta^2} - \frac{k_1 d^2}{D}C = \frac{d^2}{\lambda D}u_x\frac{\partial C}{\partial \xi} \tag{19}$$

Assuming that there is no absorption at the walls, the boundary conditions for the concentration C are

$$\frac{\partial C}{\partial \eta} = 0 \ \text{ for }\ \eta = \pm H = \pm\left[1 + \varepsilon\sin(2\pi\xi)\right] \tag{20}$$

where $\varepsilon = \dfrac{a}{d}$ is the amplitude ratio.

Assuming that $\dfrac{\partial C}{\partial \xi}$ is independent of η at any cross section and solving (19) under the boundary conditions (20), the solution for the concentration of the solute C is given as

$$C(\eta) = \left[\frac{1}{\alpha^2}\frac{d^4}{2\lambda\mu D}\frac{\partial C}{\partial \xi}\frac{\partial p}{\partial x}(1 + \lambda_1)\right]\left[\frac{2H\cosh(\alpha\eta)}{\alpha\sinh(\alpha H)} - \left(\eta^2 - \frac{H^2}{3} + \frac{2}{\alpha^2}\right)\right] \tag{21}$$

where $\alpha = \left(\dfrac{k_1 d^2}{D}\right)^{\frac{1}{2}}$

The volumetric rate Q at which the solute is transported across a section of the channel of unit breadth is defined by,

$$Q = \int_{-H}^{H} Cu_x\, d\eta \tag{22}$$

Substituting (18) and (21) in (22), we get the volumetric rate Q as

$$Q = -2\frac{d^6}{\lambda\mu^2 D}\frac{\partial C}{\partial \xi}\left(\frac{\partial p}{\partial x}\right)^2 F(\xi, \alpha, \varepsilon, \lambda_1) \tag{23}$$

where

$$F(\xi, \alpha, \varepsilon, \lambda_1) = \frac{(1 + \lambda_1)^2}{\alpha^2}\left[\frac{H}{3\alpha^4}\left(3\alpha H\coth(\alpha H) - (3 + H^2\alpha^2) + \frac{H^5}{45}\right)\right] \tag{24}$$

Comparing (23) with Fick's first law of diffusion, we find that the solute is dispersed relative to a plane moving with the mean speed of the flow with an effective dispersion co-efficient D^* given by

$$D^* = 2\frac{d^6}{\mu^2 D}\left(\frac{\partial p}{\partial x}\right)^2 F(\xi, \alpha, \varepsilon, \lambda_1) \tag{25}$$

Let the average of F be $\overline{F}$, and is defined by

$$\overline{F} = \int_0^1 F(\xi, \alpha, \varepsilon, \lambda_1)\, d\xi \tag{26}$$

2.2 Diffusion with Combined Homogeneous and Heterogeneous Chemical Reactions

We now discuss the problem of diffusion with a first-order irreversible chemical reaction taking place both in the bulk of medium (homogeneous) as well as at the walls (heterogeneous) of the channel which are assumed to be catalytic to chemical reaction. The simplified diffusion equation is same as (16).

i.e., $\dfrac{\partial C}{\partial t} + u\dfrac{\partial C}{\partial x} = D\dfrac{\partial^2 C}{\partial y^2} - k_1 C$ $\tag{27}$

The differential material balance at the walls as in Philip and Chandra [29] gives the boundary conditions as

$$\frac{\partial C}{\partial y} + fC = 0 \ \text{ at }\ y = h = \left[d + a\sin\frac{2\pi}{\lambda}(x - \bar{u}t)\right] \tag{28}$$

$$\frac{\partial C}{\partial y} - fC = 0 \ \text{at}\ y = -h = -\left[d + a\sin\frac{2\pi}{\lambda}(x - \bar{u}t)\right] \tag{29}$$

where fC gives the surface reaction rate parameter.

If we introduce the dimensionless variables (17), the diffusion equation remains as (19) and the boundary conditions become

$$\frac{\partial C}{\partial \eta} + \beta C = 0 \ \text{ at }\ \eta = H = \left[1 + \varepsilon\sin(2\pi\xi)\right] \tag{30}$$

$$\frac{\partial C}{\partial \eta} - \beta C = 0 \ \text{ at }\ \eta = -H = -\left[1 + \varepsilon\sin(2\pi\xi)\right] \tag{31}$$

where $\beta = fd$ is the heterogeneous reaction rate parameter corresponding to catalytic reaction at the walls.

The solution of (19) under the boundary conditions (30) and (31) is,

$$C(\eta) = \left[\frac{1}{\alpha^2}\frac{d^4}{2\lambda\mu D}\frac{\partial C}{\partial \xi}\frac{\partial p}{\partial x}(1 + \lambda_1)\right]\left[\frac{\cosh(\alpha\eta)}{L}\left(2H + \beta\left(\frac{2H^2}{3} + \frac{2}{\alpha^2}\right)\right)\right.$$
$$\left. - \left(\eta^2 - \frac{H^2}{3} + \frac{2}{\alpha^2}\right)\right] \tag{32}$$

where $L = \alpha\sinh(\alpha H) + \beta\cosh(\alpha H)$

The volumetric rate Q at which the solute is transported across a section of the channel of unit breadth is defined by,

$$Q = \int_{-H}^{H} Cu_x\, d\eta \tag{33}$$

Substituting (18) and (32) in (33), we get

$$Q = -2\frac{d^6}{\lambda\mu^2 D}\frac{\partial C}{\partial \xi}\left(\frac{\partial p}{\partial x}\right)^2 G(\xi,\alpha,\beta,\varepsilon,\lambda_1) \tag{34}$$

where

$$G(\xi,\alpha,\beta,\varepsilon,\lambda_1) = \frac{(1+\lambda_1)^2}{\alpha^2}\left[\left(\frac{1}{3L\alpha^3}\right)\left(H + \frac{\beta H^2}{3} + \frac{\beta}{\alpha^2}\right)\right.$$

$$\left.\left(3\alpha H\cosh(\alpha H) - (3 + H^2\alpha^2)\sinh(\alpha H)\right) + \frac{H^5}{45}\right] \tag{35}$$

Comparing (34) with Fick's first law of diffusion, we find that the solute is dispersed relative to a plane moving with the mean speed of the flow with an effective dispersion co-efficient D^* given by

$$D^* = 2\frac{d^6}{\mu^2 D}\left(\frac{\partial p}{\partial x}\right)^2 G(\xi,\alpha,\beta,\varepsilon,\lambda_1) \tag{36}$$

Let the average of G be $\overline{G}$, and is defined by

$$\overline{G} = \int_0^1 G(\xi,\alpha,\beta,\varepsilon,\lambda_1)d\xi \tag{37}$$

3   RESULTS AND DISCUSSIONS

The effects of various parameters on the average effective dispersion coefficient can be observed through the functions $\overline{F}$ and $\overline{G}$ given by equations (26) and (37) respectively. The expressions for $\overline{F}$ and $\overline{G}$ have been obtained by numerical integration using MATHEMATICA software for different values of relevant parameters and presented graphically. The important parameters involved in the expressions are: the amplitude ratio ε, the homogeneous reaction rate parameter α, the heterogeneous reaction rate parameter β and the Jeffrey parameter λ_1.

3.1 Homogeneous Chemical Reaction

Figs. 2 and 3 show the effect of amplitude ratio on effective dispersion co-efficient for different values of Jeffrey parameter, in the case of homogeneous chemical reaction. It can be noticed that the average effective dispersion co-efficient $\overline{F}$

Fig. 2 Effect of λ_1 on $\overline{F}$ (α =1)

increases with amplitude ratio ε. This may mean that peristalsis enhances dispersion.

Fig. 3 Effect of λ_1 on $\overline{F}$ (α =2)

Fig. 4 Effect of λ_1 on $\overline{F}$ (ε =0.2)
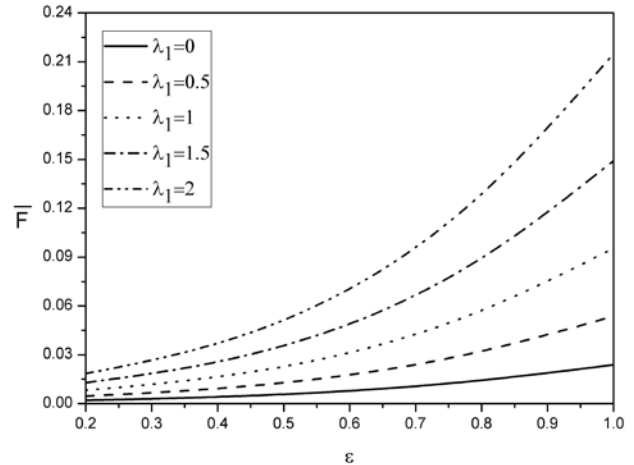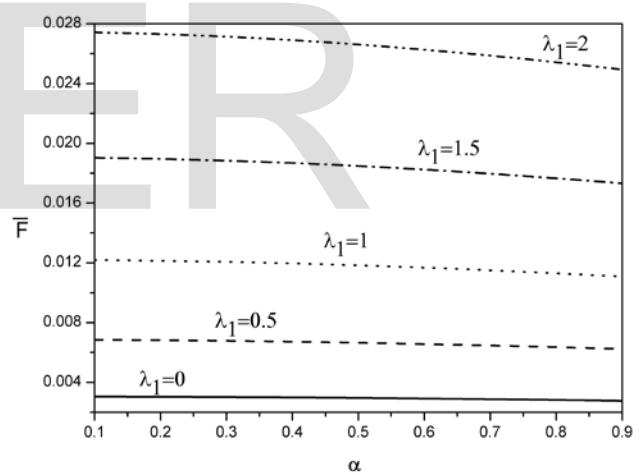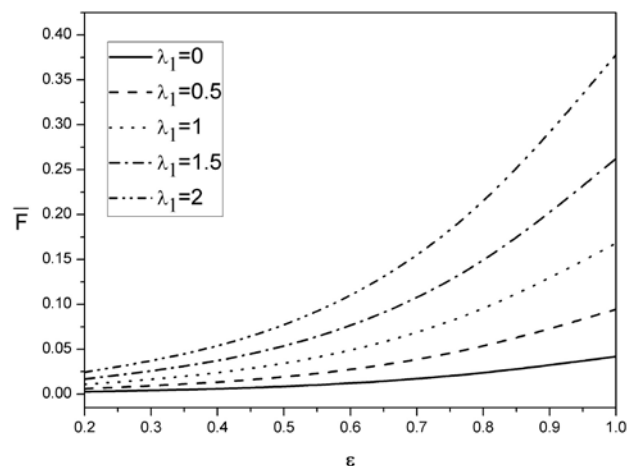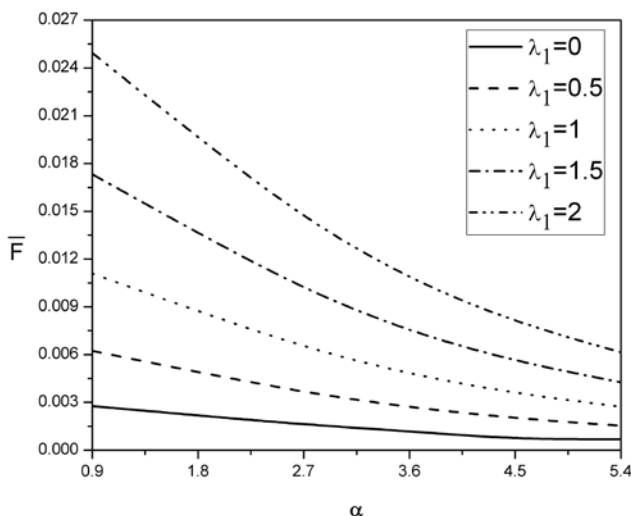
Fig. 5 Effect of λ_1 on $\overline{F}$ (ε =0.2)

The effect of Jeffrey parameter λ_1 on the average dispersion co-efficient is shown in Figs. 2 - 6. It can be observed that the dispersion increases with Jeffrey parameter λ_1.   It is also observed that the average dispersion coefficient decreases with chemical reaction parameter α for all values of amplitude ratio and Jeffrey parameters (Figs. 4 - 6). This result agrees with that of Gupta and Gupta [23], Dutta et al. [26], Ramana Rao and Padma [24], [25], Padma and Ramana Rao [22]. This result is expected since increase in α leads to increasing number of moles of solute undergoing chemical reaction and this results in the decrease of dispersion.
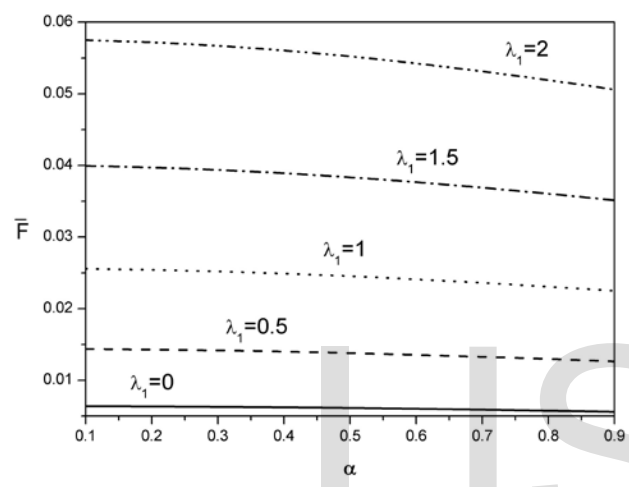

Fig. 6 Effect of λ_1 on $\overline{F}$ ($\varepsilon = 0.2$)

3.2 Combined Homogeneous and Heterogeneous Chemical Reactions

Figs. 7 – 10 display the effects of various parameters on the average effective dispersion co-efficient $\overline{G}$ for the case of combined first order chemical reactions both in the bulk and at the walls. Fig. 7 shows that the average effective dispersion co-efficient $\overline{G}$ represented by equation (37) increases with amplitude ratio ε. This implies that peristalsis enhances dispersion of a solute in fluid flow. This result is same as that obtained in the case of homogeneous reaction.
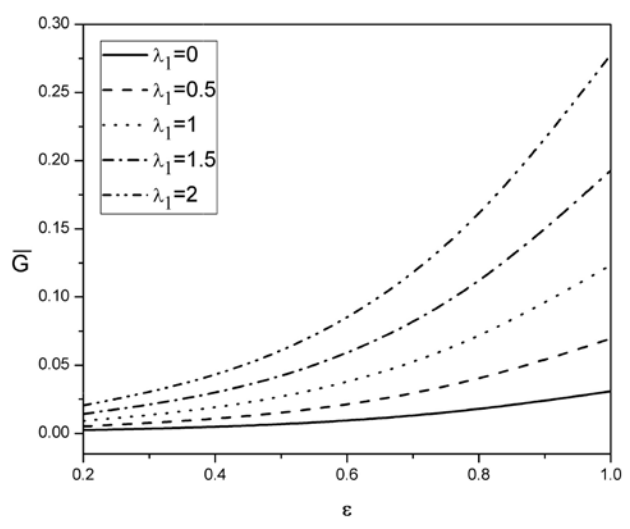

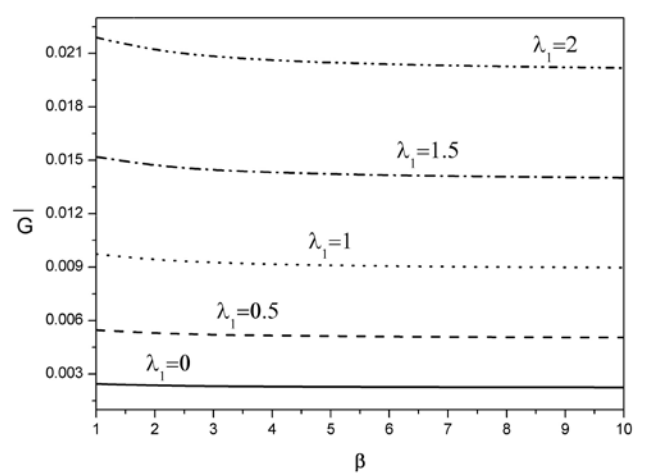Fig. 7 Effect of λ_1 on $\overline{G}$ (α =1 and β =5)

Fig. 8 Effect of λ_1 on $\overline{G}$ (α =1 and ε =0.2)

Fig. 9 Effect of λ_1 on $\overline{G}$ (α =1 and ε =0.4)

Fig. 10 Effect of λ_1 on $\overline{G}$ (β =5 and ε =0.2)

$$\theta = \frac{\iota}{\bar{t}}$$
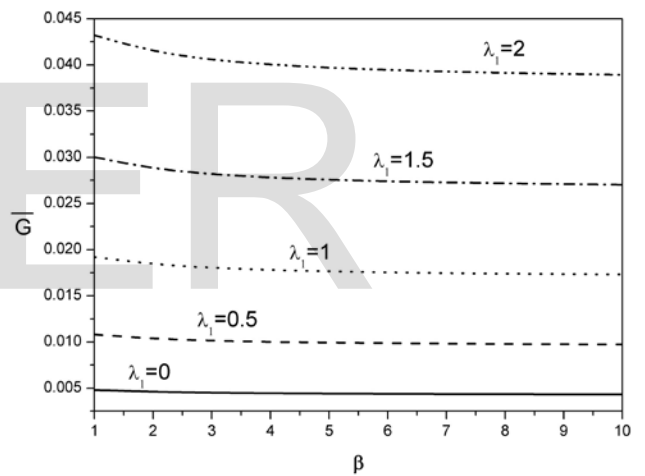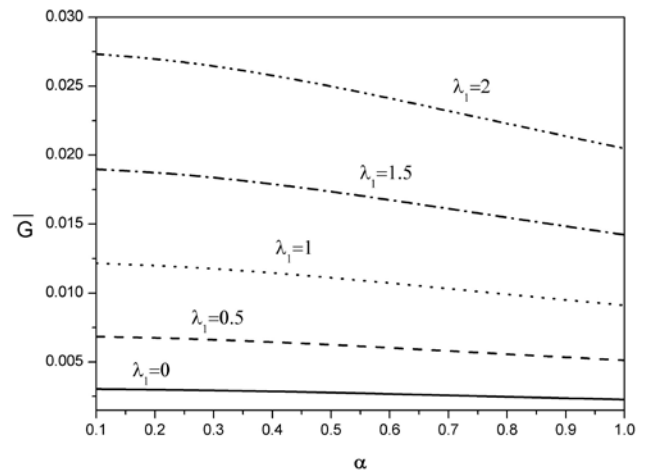
...an be seen that average dispersion co-efficient $\bar{G}$ in-...with Jeffrey parameter λ_1 (Figs. 7 - 10). This result ...with that of Alemayehu and Radhakrishnamacharya ...[...], [...]. It is also observed that the average dispersion coefficient decreases with homogeneous chemical reaction rate parameter α (Fig. 10) and heterogeneous chemical reaction rate parameter β (Figs. 8 and 9).

4 CONCLUSION

The dispersion of a solute in peristaltic motion of a Jeffrey fluid with both homogeneous and heterogeneous chemical reactions has been studied under long wavelength approximation and Taylor's limiting condition. It is observed that peristaltic motion enhances dispersion and dispersion increases with Jeffrey parameter λ_1 in the cases of both homogeneous and combined homogeneous and heterogeneous chemical reactions. Further, average dispersion co-efficient decreases with homogeneous reaction rate parameter α heterogeneous reaction rate parameter β.

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